Study on Order Policy of Two-Way Substitutable Products for a Single Retailer

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Abstract: For plenty of substitutable products commonly exist in markets, a retailer who wants to strengthen its competition has to manage the order quantity and inventory strategies scientifically. Setting the price and rate of substitution in a different way from previous researches, and studies retailer's order strategies of substitutable product is studied. At last, the study reveals how parameters, such as retail price, wholesale price, substitute rate and capital restriction, affect the order quantity.

Keywords: Retailer; two-way substitution; substitutable products; order policy

I. Introduction

In reality, a lot of products have their substitutes for the same use value. So called substitute is one product that could realize the same use value of another product. It means their relationship is competitive, i. e., increase in the sale volume of one product will decrease the substitute and vice verse, such as pork and beef. According to economic studies, for two-way substitutable products, to decrease one product's price will increase its sale volume and decrease sale volume of the other. An investigation demonstrates that only 12 to18 percent customers will hold to their loyal products when confronting stockout. For the most customers, they definitely will choose equivalent substitutes. Hence, the substitution is common and practical in the market.

In a market, product substitutability comes from use value. It demonstrates substitutable ability of products, which can be measured by substitution rate based on distinctive degree of use value. The substitution rate is a statistical value from a large number of investigations and statistics. If two products are totally different in use value, which means their distinction is infinite and substitution rate is zero, otherwise we can regard them as non-substitutable goods. Conversely, if two products are identical in use value, their distinction is zero and substitution rate is infinite, namely they are completely substitutable. Between these two extreme situations, substitution rate varies in inverse proportion as distinction degree is changing.

According to the demand characteristic of substitution, we can sort substitutes into two categories: price-driven substitution and inventory-driven substitution. The former can be described that the demand of product 2 changes when price of product 1 raise or lower, and the latter one means customers will purchase product 2 instead of product 1 when product 1 is out of stock and product 2 remains. Therein, inventory-driven substitution includes customer-driven and supplier-driven. For example, in the sale process of manufacturing products and electronics, when the supply of high-end products is surplus and low-end products is stock out, retailers will make up low-end markets with high-end products by substitution strategies and markdown promotion. At this time, inventory-driven substitution is driven by the supplier. This strategy is not suitable for perishable goods with fixed lifecycle, but satisfies customers' need passively. That means the substitution is driven by customers.

In the market almost all kinds of products have their substitutes for the same or similar use value. They are substitutable reciprocally to satisfy customers need. Practice shows that most customers will choose other products which have similar use value when confronting stockout of loyal products. Though product diversification can enhance customer satisfaction, it will also aggravate surplus or stockout. Therefore, to improve retailers' profit and satisfying customers' demand, it is significant to study and highlight order strategies of substitutable products.

II. Literature Review

Recently, the study on substitutable supply chain management has become a hotspot. Pentico（1974）studied the optimal inventory strategy of multi-product by using dynamic programming method on the basis of one-way substitution[9]. Chand, et al（1994）generalized Pentico's price function, assumed fixed demand and one-way substitution among parts, and calculated the optimal inventory portfolio by dynamic programming method[5]. Anupindi, et al（1998）established a model that considered customers would purchase substitutes, and estimated final demands of products[1]. Kouvelis, E.（1999）studied a retailer's inventory model with two substitutable products by news-vendor model[7]. These two products are indirectly substitutable, i.e., the combination of two products is a substitute of one product in it, and they are substitutable in volume.

Smith and Agrawal, et al（2000）explored optimal strategies for substitutable products with different brands and styles on the assumption of fixed service level and limited resource, so as to get the optimal profit[10]. Cai, et al (2003a) studied how a retailer make his optimal order strategy when two products are one-way substitutable, gave and proved some conditions and characters to satisfy the strategy, and demonstrated that substitution could improve retailer's profit and service level after compared to nonsubstitution strategy[3]. Chen(2004) studied order strategy of substitutable perishable products with stochastic demand, and discussed how the optimal order quantity and parameters effect the order strategy disregarding of salvage value and stockout loss[6].

Parlar(1988) discussed how two decision-makers game when their products are sold out and find the Nash equilibrium solution[8]. Zheng(1999) established a multi-product and multi-stage inventory model from the perspective of the retailer with substitution to maximize the retailer's profit, resolved how to use inventory satisfy customers' needs with considering income, salvage value, inventory cost, purchase cost, stockout loss and so on, and proved that optimal order quantity exists and be unique without providing calculation process or specific methods [13]. Bassok et al (1999) studied a multi-product inventory model with one-stage and downward substitution, and considered substitution cost without calculating process or specific methods[2]. Su(2005) studied on the application of substitute inventory model with one-stage and two-player game, and compared different order strategies under different behavior patterns of players[11]. Zhou(2007) researched single-period optimal order strategy with substitute, established two different inventory models with two-way substitutes, and gave that retailer's expect profit function is the necessary condition of concave and submodular function[12]. At last, he proved conditions the optimal order quantity should satisfy.

It must be pointed out that previous researches assumed substitutes are sold at original products' price [3]. But this assumption is not practical, especially in one-way substitution. When a product is substituted, it doesn't mean its price is substituted too. Hence, in this paper, it is more practical to assume price of the substitute is sold at its original price. Furthermore, we should distinguish price-driven substitution and inventory-driven substitution by models. Early studies, such as [3],[8], barely considered the type of substitution, and described them as product 1 substitutes product 2 at the same amount. Actually, inventory-driven substitution is brought about by stockout of product 2, so it is more reasonable that product A satisfies stockout of product B in a certain proportion, namely substitution rate can describe this proportion. Therefore, the order model the study brought out is more reasonable.

III. Order Policy for a Two-way Substitutable Products Retailer

Assume that a retailer sells two kinds two-way substitutable products. Price of each product is decided by its demand, which is stochastic. The retailer must decide how to order and find out what can affect order strategy so as to maximize his profit. This problem is analyzed as following.

Parameters and assumptions

X_i is the demand of product i, $i=1,2$. (X_1, X_2) is a two-dimensional random variable, of which joint density function is $f(x_1, x_2)$, joint probability distribution function is $F(x_1,x_2)$; in addition, the marginal density function of X_1 is $f(x_1)$, the marginal distribution function is $F(x_1)$, and the marginal density function of X_2 is $g(x_2)$, the marginal distribution function is $G(x_2)$.

q_i is order quantity of product i; w_i is wholesale price of product i; p_i is retail price of product i; h_i is unit holding cost of product i; s_i is unit stockout cost product i, $s_i \le p_i$; α_{ij} is the substitution rate of i to j, that is, when product j is out of stock, the rate of product i substituting product j, $0 < \alpha_{ij} < 1$.

This paper is based on the following assumptions:

1. The demand of products is stochastic, and retail price is determined by market.
2. The order quantity of products is considered to satisfy own needs; when product 2 is out of stock, product 1 is in stock, product 1 will be used to fulfill the demand of product 2.
3. If product 1 substitutes product 2, it will be sold at the price of product 1.
4. Products' inventory is zero at the beginning of period.

Order strategy of two-way substitutable products without capital limitation

There are four possibilities when the retailer decides order quantity of two kind of products that may or may not satisfy demand.

(1) When $q_1 \le X_1$, $q_2 \le X_2$, order quantity of two products is smaller than demand, and they don't substitute reciprocally. Then their stock is zero. Retailer's profit is equivalent to sales income subtracting stockout loss as follows:

$$\Pi_1 = (p_1 - w_1)q_1 + (p_2 - w_2)q_2 - s_1(X_1 - q_1) - s_2(X_2 - q_2) \quad (1)$$

(2) When $q_1 > X_1, q_2 \le X_2$, product 2 is out of stock, and its excess demand will be satisfied by product 1 at a substitution rate α_{12}. At this time, product 2 is sold out and needs $X_2 - q_2$ more. Product 1 remains and has $q_1 - X_1$ surpluses. The substitution rate of product 1 to 2 is α_{12}. It means $\alpha_{12}(X_2 - q_2)$ units product 1 are needed to substitute the stockout of product 2, so the actual amount substitute of product 1 is $\min[q_1 - X_1, \alpha_{12}(X_2 - q_2)]$. After substitution, product 1 has stock of $\max[(q_1 - X_1) - \alpha_{12}(X_2 - q_2), 0]$, or

product 2 has stockout of $X_2 - q_2 - (q_1 - X_1)$. Therefore, retailer's profit is

$$\Pi_2 = p_1 X_1 - w_1 q_1 + (p_2 - w_2) q_2 - s_2 \cdot \max[X_2 - q_2 - (q_1 - X_1), 0] + p_1 \cdot \min[q_1 - X_1, \alpha_{12}(X_2 - q_2)] - h_1 \cdot \max[(q_1 - X_1) - \alpha_{12}(X_2 - q_2), 0] \quad (2)$$

(3) When $q_1 > X_1$, $q_2 > X_2$, the order quantity of two products is larger than demand. There will be no substitution and stockout, and two products remain. Then the profit of retailer is

$$\Pi_3 = p_1 X_1 - w_1 q_1 + p_2 X_2 - w_2 q_2 - h_1 (X_1 - q_1) - h_2 (X_2 - q_2) \quad (3)$$

(4) When $q_1 \le X_1, q_2 > X_2$, product 1 is out of stock, and product 2 will satisfy its needs at a substitution rate of α_{21}. The status of profit is opposite to (1) as following.

$$\Pi_4 = p_2 X_2 - w_2 q_2 + (p_1 - w_1) q_1 - s_1 \cdot \max[X_1 - q_1 - (q_2 - X_2), 0] + p_2 \cdot \min[q_2 - X_2, \alpha_{21}(X_1 - q_1)] - h_2 \cdot \max[(q_2 - X_2) - \alpha_{21}(X_1 - q_1), 0] \quad (4)$$

Therefore, the profit of retailer can be summarized as

$$\Pi_r(q_1, q_2) = \begin{cases} (p_1 - w_1) q_1 + (p_2 - w_2) q_2 - s_1 (X_1 - q_1) - s_2 (X_2 - q_2), & q_1 \le X_1 \quad q_2 \le X_2 \\ p_1 X_1 - w_1 q_1 + (p_2 - w_2) q_2 - s_2 \cdot \max[\alpha_{12}(X_2 - q_2) - (q_1 - X_1), 0] & \\ \quad p_1 \cdot \min[q_1 - X_1, \alpha_{12}(X_2 - q_2)] - h_1 \cdot \max[(q_1 - X_1) - \alpha_{12}(X_2 - q_2), 0], & q_1 > X_1 \quad q_2 \le X_2 \\ p_1 X_1 - w_1 q_1 + p_2 X_2 - w_2 q_2 - h_1 (X_1 - q_1) - h_2 (X_2 - q_2), & q_1 > X_1 \quad q_2 > X_2 \\ p_2 X_2 - w_2 q_2 + (p_1 - w_1) q_1 - s_1 \cdot \max[\alpha_{21}(X_1 - q_1) - (q_2 - X_2), 0] & \\ \quad p_2 \cdot \min[q_2 - X_2, \alpha_{21}(X_1 - q_1)] - h_2 \cdot \max[(q_2 - X_2) - \alpha_{21}(X_1 - q_1), 0], & q_1 \le X_1 \quad q_2 > X_2 \end{cases}$$

The expectation of which is:

$$\begin{aligned} E[\Pi_r(q_1, q_2)] &= (p_1 + s_1 - w_1) q_1 + (p_2 + s_2 - w_2) q_2 - s_1 \cdot EX_1 - s_2 \cdot EX_2 \\ &+ \int_0^{q_1} \int_0^{q_2} [h_1 \alpha_{12}(q_2 - x_2) + h_2 \alpha_{21}(q_1 - x_1)] f(x_1, x_2) dx_2 dx_1 \\ &+ \int_0^{+\infty} \int_0^{q_2} s_2 (x_2 - q_2) f(x_1, x_2) dx_2 dx_1 + \int_0^{q_1} \int_0^{+\infty} s_1 (x_1 - q_1) f(x_1, x_2) dx_2 dx_1 \\ &- \int_0^{q_1} \int_0^{q_2 + \frac{q_1 - x_1}{\alpha_{12}}} [(p_1 + h_1)(q_1 - x_1) + h_1 \alpha_{12}(q_2 - x_2)] f(x_1, x_2) dx_2 dx_1 \\ &- \int_0^{q_1 + \frac{q_2 - x_2}{\alpha_{21}}} \int_0^{q_2} [(p_2 + h_2)(q_2 - x_2) + h_2 \alpha_{21}(q_1 - x_1)] f(x_1, x_2) dx_2 dx_1 \end{aligned} \quad (5)$$

We can prove that $E\Pi_r$ has the only optimal solution (q_1^*, q_2^*), the process of which is showed in appendix. We can solve equations (6) with known demand distribution function and some parameters to get the optimal order quantity.

$$\begin{cases} 0 = \dfrac{\partial E\Pi}{\partial q_1} = p_1 + s_1 - w_1 - s_1 F(q_1) + \alpha_{21} h_2 F(q_1, q_2) \\ -(p_1 + h_1) F(q_1, q_2 + \dfrac{q_1 - x_1}{\alpha_{12}}) - \alpha_{21} h_2 F(q_1 + \dfrac{q_2 - x_2}{\alpha_{21}}, q_2) \\ -\dfrac{p_1}{\alpha_{12}} \int_0^{q_1} (q_1 - x_1) f(x_1, q_2 + \dfrac{q_1 - x_1}{\alpha_{12}}) dx_1 \\ -\dfrac{p_2}{\alpha_{21}} \int_0^{q_2} (q_2 - x_2) f(q_1 + \dfrac{q_2 - x_2}{\alpha_{21}}, x_2) dx_2 \\ 0 = \dfrac{\partial E\Pi}{\partial q_2} = p_2 + s_2 - w_2 - s_2 G(q_2) + \alpha_{12} h_1 F(q_1, q_2) \\ -(p_2 + h_2) F(q_1 + \dfrac{q_2 - x_2}{\alpha_{21}}, q_2) - \alpha_{12} h_1 F(q_1, q_2 + \dfrac{q_1 - x_1}{\alpha_{12}}) \\ -\dfrac{p_2}{\alpha_{21}} \int_0^{q_2} (q_2 - x_2) f(q_1 + \dfrac{q_2 - x_2}{\alpha_{21}}, x_2) dx_2 \\ -\dfrac{p_1}{\alpha_{12}} \int_0^{q_1} (q_1 - x_1) f(x_1, q_2 + \dfrac{q_1 - x_1}{\alpha_{12}}) dx_1 \end{cases} \quad (6)$$

Numerical example

Assume the demand joint probability density function of product 1 and 2 is:

$$f(x_1, x_2) = \begin{cases} \dfrac{1}{400000}, x_1 \in (600,1100) \ and \ x_2 \in (600,1100) \\ 0 \quad , \quad others \end{cases}$$

Give parameters as following:

$$p_1 = 20, w_1 = 8, s_1 = 6, h_1 = 4, \alpha_{12} = 0.35$$
$$p_2 = 30, w_2 = 15, s_2 = 10, h_2 = 7, \alpha_{21} = 0.55$$

Solve it, the optimal order quantity we can get is $(q_1^*, q_2^*) = (1198, 962)$, and the maximized profit is $\Pi_r^* = 32436.32$.

Data analysis

When there are different retail prices, order quantities of two products are showed in Table 1.Table 1 shows, when the retail price of one product is fixed, order quantities of two products are proportional to it, so does to the other one. It means retailer will increase order quantities as prices of two products raise. Because no matter which kind of product is sold out, retailer could satisfy it with another product to decrease the stockout loss. When wholesale prices differ, order quantities of two products are showed in Table 2.

Table 2 shows that, as the unit wholesale price of one product is fixed, the optimal order quantities of two products are inverse proportional to it, and proportional to another product. This conclusion is accordant with [3]. So when the wholesale price of one product raises, retailer will increase the order quantity of another product. When there are different substitution rates, order quantities of two products are showed in Table 3.

Table 3 shows that, as the substitution rate is increasing, retailer will increase order quantities of two products. It is a possible for any product to use its excess quantity to satisfy the demand of the other one's. This conclusion is the same as Chen(2004)[6].

Table 1. Order quantities as retail prices vary, data is showed as (q_1, q_2)

p_2 \ p_1	16	18	20	22	24
26	1142,943	1167,947	1186,951	1200,957	1212,963
28	1151,948	1175,952	1192,957	1205,962	1216,969
30	1160,953	1181,957	1198,962	1211,967	1221,974
32	1168,958	1188,962	1204,967	1211,972	1226,979
34	1177,962	1195,966	1209,971	1216,977	1231,984

Table 2. Order quantities as wholesale prices vary, data is showed as (q_1, q_2)

w_1 \ w_2	11	13	15	17	19
4	1231,978	1238,963	1245,947	1252,931	1259,915
6	1208,985	1215,970	1222,954	1230,938	1237,922
8	1181,993	1190,978	1198,962	1206,946	1213,930
10	1153,1101	1162,986	1171,970	1179,954	1187,937
12	1121,1011	1131,995	1140,947	1150,963	1159,946

Table 3. Order quantities as substitution rates vary, data is showed as (q_1, q_2)

α_2 \ α_1	0.3	0.5	0.7
0.3	1167,912	1126,994	1078,1067
0.5	1202,905	1149,988	1091,1063
0.7	1227,900	1165,983	1098,1060

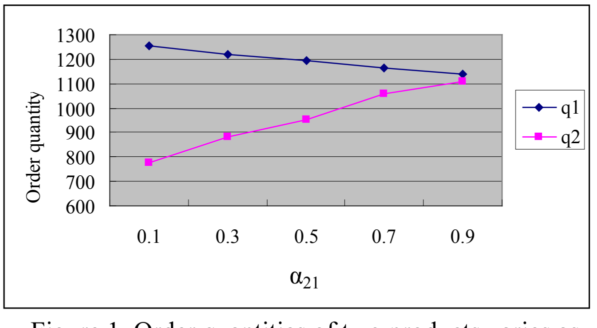

Figure 1. Order quantities of two products varies as α_{21} changes, $\alpha_{12} = 0.3$

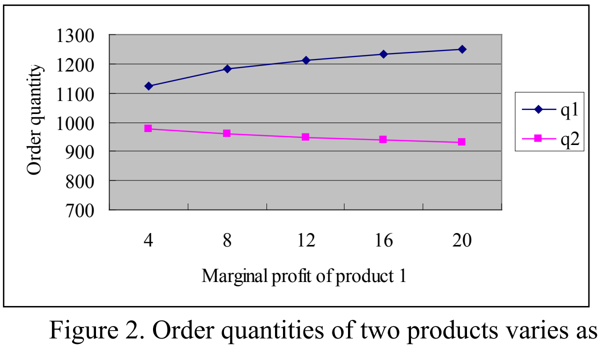

Figure 2. Order quantities of two products varies as marginal profit changes, p_2-w_2 = 10

Furthermore, we find that when substitution rate of product 1 is fixed and product 2 is raised, the order quantity of product 1 decreases slightly and product 2 increases remarkably, as shown in Figure 1. That's because as the substitution ability of product 2 to 1 improves, the relative profit of product 1 decreases, and the expected substitution profit of product 2 will increase. Hence, retailer will decrease order quantity of product 1 and increase the other one.

The retailer makes order quantity decision according to cost structure and substitution rate, but the change trend is accordant. Namely as the marginal profit increases, retailer will increase order quantity of products, as Table 4. Secondly, when the marginal profit of product 2 *p-w* keep fixed and the one of product 1 decreases, the retailer will increase order quantity of product 1 and decrease product 2. For substitution happens, substitutes are sold at their original price, and as marginal profit of product 1 increases, the relative substitution profit of product 2 to 1 will decrease. That is, when the retailer knows that it's more profitable to sell one product, he will order it at the amount that assures to satisfy demand and substitution and minimize backlog loss.

IV. Conclusions

Based on early researches, the study put forward different assumptions on the features of substitutes, established order models with two-way substitutable products, and analyzed the conclusions under different assumptions by models and numerical examples. In contrast to the former researches, the innovation of this paper is the setup of substitute price and rate.

Different form the early studies' general assumption that substitutes are sold at its original price after substituting another product, we distinguishes price-driven substitution and inventory-driven substitution by models. Early studies barely considered the differences among types of substitutions, and all of them regarded substitution as partial amount of product 1 substituting stockout of product 2 wholly. Actually, for inventory-driven substitution, it is more reasonable to describe it as proportional stockout of product 2 is satisfied by product 1, i.e., use substitution rate to reflect their relationship. Numerical examples show that order quantity is proportional to retail price, because if one product may be out of stock, its price will raise and the potential substitution profit will be improved. This conclusion is different from early studies such as Cai, et al (2003a; 2003b) [3][4], for the value of wholesale price and holding cost is different.

This paper solely studies one-stage order strategy for the retailer, the further study could expand to two-stage or even three-stage supply chain coordination. Additionally, we assume the sale happens in a single period and disregard how time would affect sales, so take time into order models will be another innovation direction.

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